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Behavior of Plain and Reinforced Concrete Slab with Slot Subjected to Uniaxial Compression

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Abstract— In this paper, studies on the behavior of plain and reinforced concrete slab with slot subjected to Uniaxial compression is carried out using the commercially available finite element (FE) tool. Initially, a plain slab with circular hole is modelled with FE tool and the results of stress profile along the directions of X&Y are plotted and the same is compared with available analytical results. Further, parametric study was carried out on plain and reinforced concrete slab with different percentage of reinforcements with full and different depth of partially penetrated rectangular slot with different aspect ratios. A total of 96 models were analyzed, and from the analysis results stress profile was plotted and stress concentration factor was evaluated. Comparison of the results obtained from the parametric studies on the stress field under the influence of the above parameters was studied.

Index Terms— Aspect ratio, Finite Element Analysis, Stress Concentration Factor, Uniaxial compression,

1 INTRODUCTION

All engineering structures are designed to be safe and durable, so as perform satisfactorily during its service life. When designing such structures, various factors have to be taken into consideration. One such factor is stress concentration. Abrupt changes in cross-section give rise to great irregularities in stress distribution. These irregularities are of particular importance in the design of structural components subjected to variable external forces and to reversal of stresses. Abrupt changes originate from irregularities, results in distribution of stresses are known as stress concentrators. An opening in general is the area of stress concentration, but is essential to meet functional requirements.

Mostly the slots are required for floor slabs, walls and other two/three dimensional elements. Cracks in structures often initiate and propagate from the locations of stress concentration. The majority of service cracks generates in the area of stress concentration around the edge of the opening. Hence knowledge of stress concentration in the vicinity of opening is essential for the reliable design of structural components, and therefore it becomes important to understand the behavior of structural members subjected to stress concentration and therefore, the complete study of stress state of structural members in general, and the concentration of stress around openings in particular, becomes the integral/essential part of the analysis.

Originally stress concentrations were worked out mathematically which was satisfactory for simple configurations. More complex configurations could only be determined by the experimental work which was time consuming and expensive. The development of FE method made it possible to compute stress concentration factors easily. The stress concentration factor can be evaluated by using computational techniques, elasticity theory and the experimental methods. Peterson (1961)[1] has provided stress concentration factors for different geometric irregularities under various types of loadings. Many researchers have contributed in this area by considering different opening geometry and loading pattern. Studies on the stress field on structural members with slot are very limited and very few reported literature is available. Hence, an attempt has been made to understand the stress field pattern of the slab with slot subjected to uniaxial compression. This paper presents the details on the analytical studies carried out on a plain and reinforced concrete slab with fully and partially penetrated slot with different aspect ratio. Further a numerical model in ABAQUS is developed similar to the analytical studies available in the literature using finite element software and the results, is validated. Further, parametric study was carried out on plain and reinforced concrete slab with different percentage of reinforcements with full and different depth of partially penetrated rectangular slot with different aspect ratios. A total of 96 models were analyzed, and from the analysis results stress profiles were plotted and stress concentration factor was evaluated. Comparison of the results obtained from the parametric studies on the stress field under the influence of the above parameters was studied.

2. BACKGROUND

Recent research by Hasan, A. Das, T. Rahman (2009) investigated the effects of hole/slot size, shape and orientation on the stress-strain distribution. Finite element method is used for the solution of two-dimensional elastic plates incorporating the hole located at the center of the plate. Results from finite element method have been compared with the analytical results for different shapes. Finite element results, is carried out by using the commercial software COMSOL 3.3. For the elliptical hole, at 0° angular position of the elliptical hole/ slot, the maximum stress occurs at the two ends of the hole on its minor axis. At 90° angular position of the elliptical hole, the maximum stress occurs at the two ends of the hole on its major axis. For elliptical hole, with the increase of plate length to width ratio the maximum stress at all angular position increases. Square hole experiences more longitudinal stress than the elliptical hole. If the holes are made to rotate, the value of the maximum stress increases with rotation.

The investigation on the effect of meshes in the analysis results have been carried out by M. V. Kavade (2013) by analyzing a plate with a circular the hole subjected to a uniform stress with various meshes. The different types of meshes used are Auto mesh, Ruled mesh, Spline option, Spin option, manual meshing, Split element options and Edit element. The obtained values are shown in

Table 1. A lot of Variations in the results is obtained through various meshes. The FEA results are significantly affected by the meshing algorithm and this is reflected in the results. The ruled mesh (Middle Ruled Mesh without Washer) which is symmetric offers a symmetric response in the plots whereas the other meshes don't offer that symmetry of the solution to a desirable extent.

Table I Maximum Stress and Displacement Due to Different Meshing in the Analysis By M.V Kavade

sr.no.	Title	max.disp	max.stress
1	Plate With Hole Automesh	4.015e-05	5.745e+02
2	Corse Ruled Mesh Without Washer	4.086e-05	6.073e+02
3	Fine ruled mesh without washer	4.167e-05	8.209e+02
4	Middle Ruled Mesh Without Washer	4.037e-05	7.767e+02
5	Single Washer	2.401e-04	4.729e+02
6	Double washer	2.475e-05	5.309e+02
7	Three Washer	2.582e-05	5.827e+02

M. Arifur Rahman (2005) explained the application of finite difference technique for the solution of an elastic problem of a rectangular the body having an internal circular the hole. They also focused on the stress concentration developed at the critical zones around the circular the hole of the body. The results found for the slot boundary had good agreement with the analytical solution. The solution approach used will be valid for any arbitrary shape of the body having arbitrary shaped an internal hole/slot.

G. H. Lei (2001) presented a simple approximate solution to the problem for infinite elastic and an isotropic plate with the rectangular hole subjected to uniaxial stress at infinity. A linear elastic plane stress analysis was carried out using complex variables. To investigate the accuracy of the solutions derived in the paper, stresses and displacements around a rectangular hole of 3x1 m, subjected to a remote uniaxial stress q at $\alpha = \pi/2$ and $\alpha = 0$, respectively, were calculated by using both derived solutions and finite-element package SAP2000. A total of 17,600 elements were generated in a quarter plate of 50x50 m in the finite-element analyses using SAP2000.

Reissner and Morduchow (1949) considered a neutral circular hole. They allowed themselves the choice of the tensile stiffness and bending stiffness of the reinforcement. Apart from a few particular loading systems their solutions were impracticable, mainly because the required bending stiffness was so large as to be practically incompatible with the required tensile stiffness. They concluded that for the types of reinforcement deemed technically important the bending stiffness may be ignored or in other words, the reinforcement around a neutral hole experiences primarily tensile or compressive loads rather than bending moments.

Study on the effect of slot/hole size and shape on stress distribution was carried out by Jansher Sadik. The Parametric study was conducted to examine the sensitivity of the stress concentration factor and the influence of different percentage of reinforcement and its location on the slab with respect to the slot. A numerical model in ABAQUS was developed similar to the analytical study using finite element software. Results from finite element method have been compared with the analytical results for different shapes of slots. It was observed that the percentage of reinforcement in a concrete slab plays a very significant role in the stress concentration factor (SCF). The location of the maximum stress concentration factor moves away from the mid plane of the slab as the thickness increases. Significant Variations was observed in the stress concentration factor at the edge and mid location of the slot. This is due to the aspect ratio of the slot (width / thickness) as this ratio is directly proportional to the ratio of SCF at mid to the edge location. The values of the stress concentration factor are greatly affected by the location of the reinforcement with respect to the slot. Stress profile is greatly influenced by the slot size to the plate size ratio. Also, it was observed that as this ratio increases, the stress profile varies rapidly.

3. MODELLING AND THE ANALYSIS

The Finite Element Method (FEM) is the dominant discretization technique in structural mechanics. The basic concept in the physical interpretation of FEM is the, subdivision of the mathematical model into disjoint (non-overlapping) components of simple geometry called finite elements. Finite element method is one of the most powerful tools for simulating the real/actual model and the results of the FE model is very close to the test results depending on the refinement of mesh size and other factors. Further the FEM saves the time and cost in comparison to actual experiments. In this study commercially available finite element package ABAQUS is used for the study. In order to validate the FE model, a three dimensional slab with circular hole is analyzed and the results are compared with the theoretical values derived from the basic equations.

Considering a the slab element with a circular hole subjected to an axial compression, and then stress Variations is calculated using equation,

$$\sigma_r = \frac{S}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{S}{2}\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{S}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{S}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = -\frac{S}{2}\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\sin 2\theta$$

Where,

S - Applied axial compression

a - radius of circular the hole,

r - Distance between the analyzing point and center of the hole.

From these equations σx or the stress at the loading direction can be calculated as,

$$\sigma_x = \sigma_r \cos^2\theta + \sigma_\theta \sin^2\theta - 2\,\tau_{r\theta} \sin\theta\cos\theta$$

From the above formulas it is evident that the maximum stress occurs at the end of the holes perpendicular to the direction of compression. While substituting r= a and $\theta = \pi/2$ and $3\pi/2$, the values is,

$$\sigma_r = 0,\quad \sigma_\theta = -\sigma,\quad \tau_{r\theta} = 0 \quad \text{at } r = a \text{ and } \theta = 0, \pi$$

$$\sigma_r = 0,\quad \sigma_\theta = 3\sigma,\quad \tau_{r\theta} = 0 \quad \text{at } r = a \text{ and } \theta = \pi/2, 3\pi/2$$

In order to validate the FE model, a slab of size 500mm x 500mm x 100 mm with a circular hole of 50 mm in diameter is modelled using ABAQUS (Fig. 1). The Variations of stress pattern on the slab with circular hole (FE model) under uniform compression is shown in Fig. 2. The stress concentration factor at different locations of a/r values is evaluated from the FE model and the same is validated with the analytical results. Stress concentration factors computed on the edge and center of the slab of FE model were plotted and compared with the two-dimensional analytical results (Fig. 3).

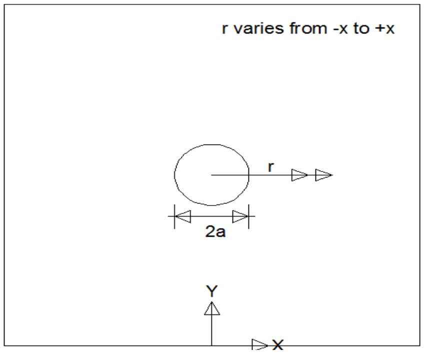

Fig. 1 Sketch showing the slab with a circular the hole

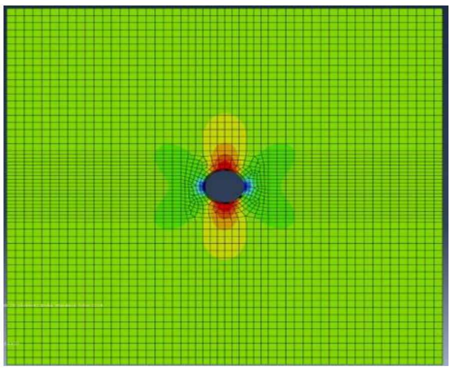

Fig. 2 Stress pattern on the slab with the hole

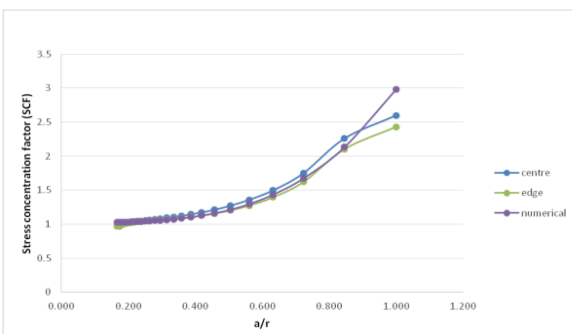

Fig. 3 Comparison of SCF with analytical study

Similar to the above, another slab of size 500mm x 500mm x 150 mm with a the rectangular the slot of size 100mm x 10mm at its center was modelled as shown in Fig. 4. The analytical results obtained from the works carried out by G.H lei is compared with the results obtained from FE study and the plot of stress concentration at the edge of the slot is shown in Fig. 5.

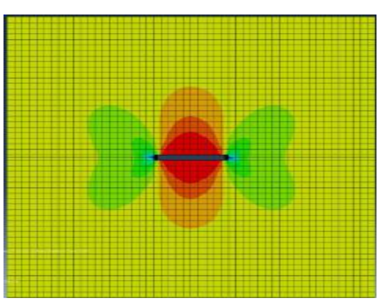

Fig. 4 Stress pattern on the slab with the rectangular the slot

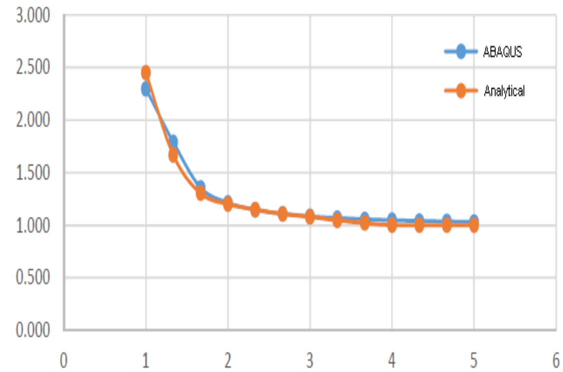

Fig. 5 Comparison of SCF with analytical study

From the above comparison, it is concluded that the results obtained using ABAQUS is found to be in line with the analytical values. From this further numerical study using ABAQUS was carried out on three dimensional slabs with full and partially penetrated slot of different dimensions and also with different percentage of reinforcements to study the stress field and also to evaluate the stress concentration factor for the above conditions.

Initially, a three-dimensional slab of size 500mm x 500mm x 100 mm (L x B x H) with the rectangular slot of dimension 100mm x 10mm (Ls x Bs), penetrated up to full slab thickness, were modelled (Fig. 6). Uniaxial compressive stress of magnitude 1 MPa is applied along the Y-Y axis of the slab. Dead load of the slab is not considered for the stress the analysis. The mechanical properties of the slab and reinforcement are given in Table 2. From the analysis, the Variations of the stress (σy) along x & y directions were plotted. From the graph, the stress concentration factor is evaluated. Further studies was carried out on the slab with full and partial penetrated the slot with different aspect ratio (Ls/Bs) and also with different percentage of reinforcement. The details of the models considered in the parametric study are given in Tables 3 to 5.

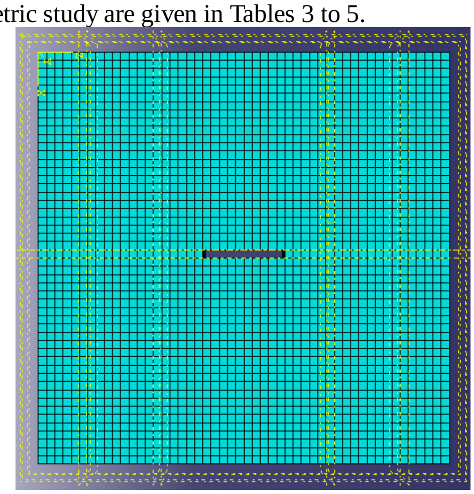

Fig. 6 FE model of the slab with the rectangular the slot

Table 2 - Mechanical Properties of Concrete and Steel Reinforcement

Material	Elastic Modulus (MPa)	Poisson's Ratio
Concrete	25000	0.17
Steel	200000	0.3

Table 3 – Different aspect ratios of the slot considered for the study.

S.No.	$L_{the\ slot}$ (mm)	$B_{the\ slot}$ (mm)	Aspect Ratio
1	100	10	10
2	80	10	8
3	60	10	6
4	40	10	4
5	20	10	2
6	10	10	1
7	10	40	0.25
8	10	100	0.1

Table 4 – Different The penetration depth of the slot considered for the study.

S.No.	Type of Penetration	The penetration depth (mm)
1	Full	100
2	Partial	30
3	Partial	50
4	Partial	70

Table 5 – Different Percentage of reinforcement considered for the study.

S.No.	Type of The slab	Reinforcement (%)
1	PCC	0
2	RCC	1
3	RCC	2

From the above different parameters considered, a total of 96 models was studied.

4. RESULTS AND DISCUSSION

The analysis of 96 models was carried out to study the influence of different aspect ratios of the slot dimension, depth of penetration and the presence of reinforcement on the stress concentration factor. In addition, behavior of the slab with slot is clearly understood with the stress profiles plotted. The results of the analysis are discussed.

Values of stress concentration factor (SCF) of the rectangular the slab with different aspect ratio varying from 0.1 to 10 for full and partial penetrated the slots is given in Table 6.

Table 6 – Stress Concentration Factors for Full and Partially Penetrated the slots

Aspect Ratio (slot length/ width)	stress concentration factors (SCF)			
	Full penetrated slot	Partially penetrated slot (30mm)	Partially penetrated slot (50mm)	Partially penetrated slot (70mm)
10	-2.337	-2.08	-1.721	-2.289
8	-2.100	-1.926	-1.652	-2.068
6	-1.859	-1.754	-1.566	-1.838
4	-1.605	-1.554	-1.451	-1.596
2	-1.325	-1.309	-1.278	-1.321
1	-1.154	-1.142	-1.134	-1.146
0.25	-1.276	-1.256	-1.233	-1.268
0.1	-1.22	-1.204	-1.186	-1.212

Similarly the values of stress concentration factor (SCF) of the rectangular the slab with different aspect ratio varying from 0.1 to 10 for full and partial penetrated the slots with different percentage of reinforcement is given in Tables 7 to 10.

Table 7 – SCF for Fully penetrated slots with different % Reinforcement

Aspect Ratio (slot length/ width)	stress concentration factors (SCF) (Fully Penetrated- PCC & RCC)		
	PCC (0%)	RCC (1%)	RCC (2%)
10	-2.337	-2.201	-2.058
8	-2.100	-1.986	-1.865
6	-1.859	-1.764	-1.662
4	-1.605	-1.526	-1.441
2	-1.325	-1.262	-1.193
1	-1.154	-1.099	-1.04
0.25	-1.276	-1.219	-1.157
0.1	-1.22	-1.166	-1.108

Table 7 – SCF for partially penetrated the slots (30mm) with different % Reinforcement

Aspect Ratio (slot length/ width)	stress concentration factors (SCF) (Partially Penetrated (30mm)- PCC & RCC)		
	PCC (0%)	RCC (1%)	RCC (2%)
10	-2.08	-1.962	-1.837
8	-1.926	-1.824	-1.716
6	-1.754	-1.667	-1.573
4	-1.554	-1.48	-1.4
2	-1.309	-1.249	-1.183
1	-1.142	-1.09	-1.033
0.25	-1.256	-1.201	-1.142
0.1	-1.204	-1.152	-1.096

Table 9 – SCF for Partially Penetrated Slots (50mm) with different % Reinforcement

Aspect Ratio (slot length/ width)	stress concentration factors (SCF) (Partially Penetrated (50mm)- PCC & RCC)		
	PCC (0%)	RCC (1%)	RCC (2%)
10	-1.721	-1.622	-1.518
8	-1.652	-1.563	-1.47
6	-1.566	-1.487	-1.403
4	-1.451	-1.381	-1.305
2	-1.278	-1.218	-1.153
1	-1.134	-1.081	-1.024
0.25	-1.233	-1.179	-1.12
0.1	-1.186	-1.134	-1.079

Table 10 – SCF for Partially Penetrated Slots (70mm) with different % Reinforcement

Aspect Ratio (slot length/ width)	stress concentration factors (SCF) (Partially Penetrated (70mm)- PCC & RCC)		
	PCC (0%)	RCC (1%)	RCC (2%)
10	-2.289	-2.157	-2.018
8	-2.068	-3.897	-3.648
6	-1.838	-3.472	-3.257
4	-1.596	-3.018	-2.836
2	-1.321	-2.501	-2.353
1	-1.146	-2.172	-2.045
0.25	-1.268	-2.403	-2.263
0.1	-1.212	-2.3	-2.17

Variations of stress concentration factor for full and partially penetrated the slots with different aspect ratio as mentioned in table 5 is shown in Fig. 7.

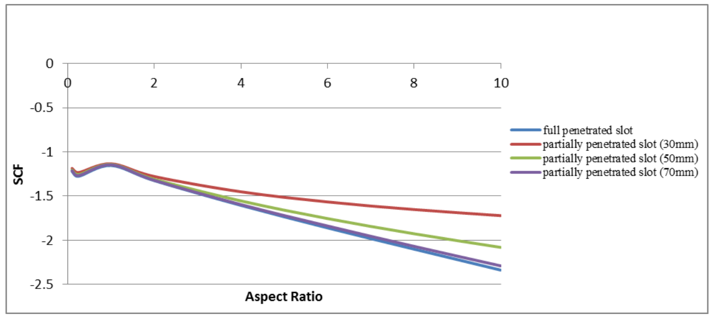

Fig. 7 Variations of SCF for full and partial penetrated the slot with different aspect ratio.

Similarly, typical Variations of stress concentration factor for full penetrated slots with different % percentage of reinforcement ratio with different aspect ratio as mentioned in table 6 is shown in Fig.8.

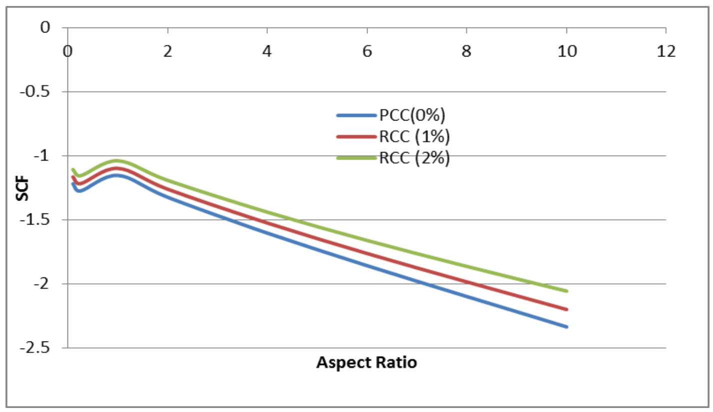

Fig. 8 Variations of SCF for full penetrated slot with different percentage of reinforcement.

In order to understand the behavior of slab with slot under uniaxial compression, in addition to the stress concentration factor, stress profiles were plotted for both full and partial penetrated slot. The Variations of the stress on the rectangular slab with fully penetrated slot along the direction of the applied stress (breadth) for different thickness location is shown in Fig.9.

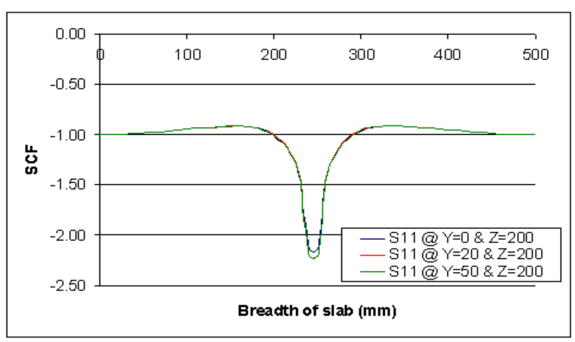

Fig. 9 Variations of stress at different locations (thickness) for full penetrated the slot (PCC).

From the above figure, it is observed that the magnitude of the stress concentration at the outer plane is less compared to the mid-plane. Similarly, to understand the Variations of stresses at different locations along the direction of the thickness for a typical partially penetrated slot (50mm) is plotted as shown in Fig. 10.

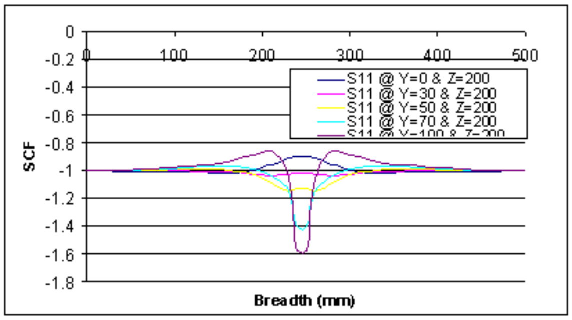

Fig. 10 Variations of stress at different locations (thickness) for Typ. Partially penetrated the slot (PCC).

From the above results it is observed that there is a significant Variation in the SCF under the influence of the dimensions of the slot, the penetration depth of the slot, presence of reinforcement. The value of SCF increases as the aspect ratio of the slot increases i.e. the dimension of the slot perpendicular to the loading direction influences greatly the value. The value of SCF is less for partially penetrated the slot when compared with the fully penetrated slot. Similarly, the SCF value for reinforced concrete element is less when compared with the plain concrete as the presence of reinforcement exhibits inherent stiffness.

5. CONCLUSION

The stress field and stress concentration factor of a plain and reinforced concrete slab with full and partially penetrated the rectangular slot subjected to uniaxial compression is systematically examined using the 3D finite element method. Parametric studies were conducted to examine the sensitivity of the stress concentration factor under the influence of different dimension of the slot, different depth of penetration of plain and reinforced concrete with different percentage of reinforcement. From the studies made, following conclusion can be drawn:

1. In case of PCC for the same penetration depth, the stress concentration factor increase as the aspect ratio increases from 1 to 10.
2. In case of fully as well as partially penetrated the slot in PCC and RCC with (1 % & 2% reinforcement), the stress concentration factor increases as the aspect ratio increases from 1 to 10.
3. In case of fully penetrated the slot in PCC and RCC with (1 % & 2% reinforcement), for the same aspect ratio, the stress concentration factor reduces from PCC to RCC respectively.

The analytical and numerically stress concentration factor for circular as well as the rectangular slot in the slab subjected to compressive load is found to be in good agreement with each other which shows the efficiency of ABAQUS software result with analytical values. So, the present study mainly concluded that FEM is the best tool to investigate the structural behavior of PCC as well as RCC specimen and it helps us to estimate various structural parameters like stress concentration factor and Variations of the stress field at different locations. This helps us to understand the behavior of the penetration depth as well as the effect of reinforcement in concrete specimen as compared to PCC. ABAQUS not only helps us to estimate SCF and the stress field for various cases without carrying out the experiment which is time consuming, but also helps us to reduce the experimental (material) cost.

6. ACKNOWLEDGEMENT

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