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Collision Dynamics of Optical Dark Solitons in a Generalized Variable-Coefficient Higher Order Nonlinear Schrödinger System from Inhomogeneous Optical Fibers

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Abstract—We consider the generalized variable-coefficient single component nonlinear Schrödinger system with higher order effects such as the third-order dispersion, self-steepening and self-frequency shift, a model equation for the propagation of intense electromagnetic field in inhomogeneous optical fibers. For describing the long-distance communication, we obtain the optical multi-dark soliton using Hirota's bilinearization method. We are able to control the characteristics of optical multi-dark solitons in inhomogeneous optical fibers by choosing suitable variable-coefficient functions.

Index Terms — dark solitons; generalized variable-coefficient nonlinear Schrödinger system; Hirota's bilinearization method; inhomogeneous optical fibers.

I. Introduction

Recently, the optical solitons have been put to use in the long-haul optical communication links and transoceanic systems to enhance confidentiality, where the required information is finally retrieved by dark-bright soliton conversion. Optical solitons are recognized as powerful laser pulses, which when propagating through optical fibers are influenced by higher order effects such as, higher order dispersion, self-steepening, stimulated inelastic scattering, and delayed nonlinear response. In this paper, we have shown that the characteristics of optical multi-dark solitons, dispersion management, and soliton amplification are controllable in communication lines.

We consider the integrable generalized variable-coefficient single component nonlinear Schrödinger system with higher order effects such as the third-order dispersion, self-steepening and self-frequency shift, as follows:

$$i\,u_z + a(z)\,u_{tt} + b(z)\,|u|^2\,u + i\,c(z)u_{ttt} + i\,d(z)(\,|u|^2\,u)_t + ie(z)\,(|u|^2)_t u + if(z)u_t + [g(z) + i\,h(z)]u \quad (1)$$

where u(z, t) is the complex envelope of the electrical field in the comoving frame, z and t, respectively, represent the normalized propagation distance along the fiber and retarded time, while all the variable coefficients are real analytic functions. a(z) and c(z) denote the group velocity dispersion and third-order dispersion, respectively. b(z) accounts for the cross-phase modulation, while d(z) is the self-steepening and e(z) is related to the delayed nonlinear response effects. The term proportional to f(z) results from the group velocity and h(z) represents the amplification or absorption coefficient.

The conditions for Eqn.(1) to be integrable were determined by the Painlevé singularity structure analysis.[1] To derive the multi-dark soliton solutions for Eqn (1), we have utilized the integrable case given below: $d(z) = -e$

$$a(z)e(z) + 3b(z)c(z) = 0;$$

$$h(z) = \frac{-e(z)c'(z) + c(z)e'(z)}{2c(z)e(z)}$$

Our aim will be to investigate the different forms of multi-dark soliton propagation in inhomogeneous optical fibers by suitably choosing the variable-coefficient functions

II. HIROTA'S BILINEARIZATION METHOD

Multi-dark soliton solutions are derived by Hirota's bilinear method[2],[3],[4],[5], since it is the most successful direct technique for constructing exact solutions to various non-linear PDEs.

As the first step, we transform U(z, t) to a quadratic form by using the following transformation:

$$u(z,t) = m(z)\frac{G(z,t)}{F(z,t)}$$

where m(z) and G(z, t)are complex functions and F(z, t) is a real one. This new form of Eqn.(1) is written in terms of 'D' operator as the combination of variable-coefficient bilinear equation.

$$3c(z)D_t^2(F.F) + e(z)|m(z)|^2|G|^2 = \lambda(z)(F.F) \qquad (2)$$

$$\left\{iD_z - 3\frac{b(z)c(z)}{e(z)}D_t^2 + ic(z)D_t^3 + [if(z) - i\lambda(z)]D_t + \frac{\lambda(z)b(z)}{e(z)}\right\}(G.F) = 0 \qquad (3)$$

Where λ(z) is a function to be determined, m(z) = $\sqrt{c(z)/e(z)}\, e^{i\int g(}$ with A≠ 0 as an arbitrary complex constant. The binary operators D_z, D_t, and are defined by [5]

$$D_z^m D_t^n(f.g) = \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n f(z,t)g(z',t')|_{z'=z,\ t'=t}$$

In the next stage, we assume power series expansion for G(z, t) and F(z, t)

$$G = g_0(1 + \chi g_2 + \chi^2 g_4 + \cdots),$$

$$F = 1 + \chi f_2 + \chi^2 f_4 + \cdots$$

When substituted to Eqn.(2) & (3), we obtain a system of equation at orders of χ which allows for determination of its coefficients by recurrence.

III. MULTI-DARK SOLITON SOLUTIONS

In sec. II, we have derived the bilinear forms of variable-coefficient nonlinear Schrödinger system. In this section, we calculate the two- and three- soliton solutions. Once the form of two- and three- soliton solutions are known, their characteristics reveal the form of Higher Order Soliton solutions.

Two-Dark Soliton Solution

In order to construct the dark two-soliton solutions, we assume

$$G = g_0(1 + \chi g_2 + \chi^2 g_4),$$

$$F = 1 + \chi f_2 + \chi^2 f_4 \qquad (4)$$

Substituting expressions (4) into Eqs. (2) and (3), and then collecting the coefficients of the like power of χ and solving the obtained equations, we have

$$g_0 = B\,[e^{i\phi_1} + e^{i\phi_2}] \qquad (5)$$

Where

$$\phi_1(z,t) = r_1 t + \int\left[r_1^3 c(z) + r_1\lambda(z) + \frac{3r_1^2 b(z)c(z)}{e(z)} + \frac{b(z)\lambda(z)}{e(z)} - r_1 f(z)\right]dz + \phi_0$$

$$\phi_2(z,t) = r_2 t + \int\left[r_2^3 c(z) + r_2\lambda(z) + \frac{3r_2^2 b(z)c(z)}{e(z)} + \frac{b(z)\lambda(z)}{e(z)} - r_2 f(z)\right]dz + \phi_0$$

and

$$f_2 = -g_2 = e^{\psi_1} + e^{\psi_2} \qquad (6)$$

$$f_4 = g_4 = \frac{1}{2}[e^{2\psi_1} + e^{2\psi_2} + 2e^{\psi_1+\psi_2}] \qquad (7)$$

With

$$\psi_1(z,t) = p\,t + \int\left[-p^3 c(z) + 3pr_1^2 c(z) + p\lambda(z) + \frac{6pr_1 b(z)c(z)}{e(z)} - pf(z)\right]dz + \psi_0$$

$$\psi_2(z,t) = p\,t + \int \left[-p^3 c(z) + 3pr_2^2 c(z) + p\lambda(z) + \frac{6pr_2 b(z)c(z)}{e(z)} - pf(z) \right] dz + \psi_0$$

where $\lambda(z) = \lambda_0\, c(z)$, with $B \neq 0$ as arbitrary complex constants, φ_0, ψ_0, $r \neq 0$, $p \neq 0$ and $\lambda_0 \neq 0$ as arbitrary real constants, and $\lambda_0 = \frac{3}{2}p^2 = |A|^2|B|^2|g_0|^2$. Now using the expressions $g_0(z,t), g_2(z,t), g_4(z,t)$, $f_2(z,t)$ and $f_4(z,t)$, the dark two-soliton solution of system (1) can be written as,

$$u(z,t) = m(z)\frac{G(z,t)}{F(z,t)} = AB_1\sqrt{\frac{c(z)}{e(z)}}\, e^{i\int g(z)dz} *$$

$$\left[\frac{(e^{i\phi_1} + e^{i\phi_2})\left[1 - (e^{\psi_1} + e^{\psi_2}) + \frac{1}{2}(e^{\psi_1} + e^{\psi_2})^2\right]}{1 + (e^{\psi_1} + e^{\psi_2}) + \frac{1}{2}(e^{\psi_1} + e^{\psi_2})^2} \right]$$

Three-Dark Soliton Solution

To construct three-dark soliton solution, we assume the power series expansion to be

$$G = g_0(1 + \chi g_2 + \chi^2 g_4 + \chi^3 g_6),$$

$$F = 1 + \chi f_2 + \chi^2 f_4 + \chi^3 f_6 \qquad (8)$$

Substituting expressions (4) into Eqs. (2) and (3), and then collecting the coefficients of the like power of χ and solving the obtained equations, we have

$$u(z,t) = m(z)\frac{(e^{i\phi_1} + e^{i\phi_2} + e^{i\phi_3})}{D}$$

$$\frac{\left[1 - (e^{\psi_1} + e^{\psi_2} + e^{\psi_3}) + \frac{1}{2}(e^{\psi_1} + e^{\psi_2} + e^{\psi_3})^2 - \frac{1}{4}(e^{\psi_1} + e^{\psi_2} + e^{\psi_3})^3\right]}{D}$$

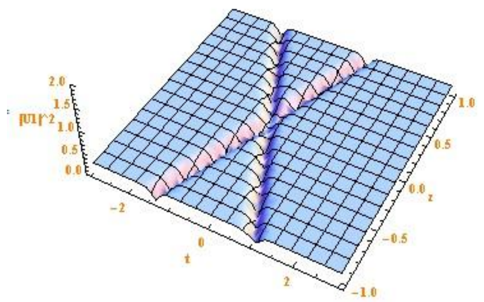
$$D = 1 + (e^{\psi_1} + e^{\psi_2} + e^{\psi_3}) + \frac{1}{2}(e^{\psi_1} + e^{\psi_2} + e^{\psi_3})^2 + \frac{1}{4}(e^{\psi_1} + e^{\psi_2} + e^{\psi_3})^3$$

IV. ANALYSIS AND APPLICATIONS

Recently, increased interest in optical dark solitons has become connected with their possible application in optical logic devices and waveguide optics as dynamic switches and junctions. They are also considered for signal processing and communication applications because of their inherent stability. The influence of noise and fiber loss on dark solitons is much lesser than that on bright solitons. Motivated by these facts and that the obtained solutions include distributed functions, one can explain different types of soliton control or dispersion management by choosing various forms of these functions.

From the expressions of two- and three- dark solitons, it is obvious that the soliton amplitude is related to the functions c(z) and e(z). So, to keep the amplitude invariant, the value of c(z) /e(z) must be a constant. Also, the soliton velocity and acceleration are related to b(z), c(z), e(z) and f(z), but not to g(z). In Figs. 1 and 6, the parameters are adopted as A = 1+3i, B = 3+4i , r = 5, $\varphi_0 = 5$, $\lambda_0 = 380$, and $p = 2\sqrt{190/3}$, while in Figs. 2–5, A = −0.5+0.8i, B = 0.6+0.4i, r = 1, $\varphi_0 = 1$, $\lambda_0 = 0.5785$, and p = 0.621 021. When b(z), c(z), e(z) and f(z) are taken some constant values, the function $\psi(z,t)$ possesses the traveling wave form and the dark soliton propagates stably as seen in Fig. 1. Figures 2–6 clearly display the evolution and propagation of the dark soliton with the effects of the distributed functions.

Figures 2–3, respectively, illustrate various stable propagations of the optical dark solitons in a fiber with distinct periodically varying inhomogeneous effects, from which we can see different influences of different distributed functions. Figures 4–5, under the effects of more distributed functions, depict the propagations of the optical dark solitons with periodic oscillation along the distance z. Compared



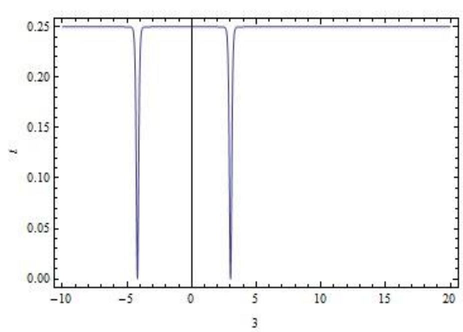


Fig. 5 with other figures, we note that the amplitude of the dark soliton moves with periodic growth and decay.

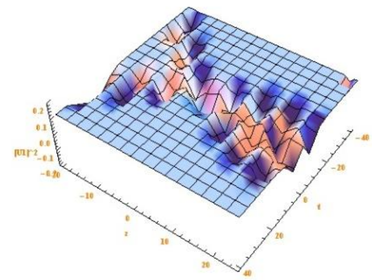


FIG. 1. The parameters adopted here are c(z) = 0.1, e(z) = 2.5, b(z) = 1 and f(z) = 1.5; (b) is the intensity plot corresponding to (a).FIG. 2. The parameters adopted here are c(z) = 1, e(z) = 2.5, b(z) = 1, and f(z)=1.5 sin(z);

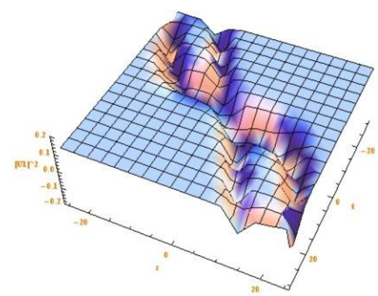


FIG. 3 The parameters adopted here are c(z) = 1, e(z) = 2.5, b(z) = 1, and f(z)=1.5 sin(z);

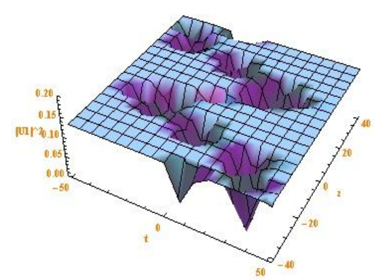


FIG.4. The parameters adopted here are c(z) = 2 Cos(z), e(z) = Cos(z), b(z) = 0.1 and f(z) = 1;

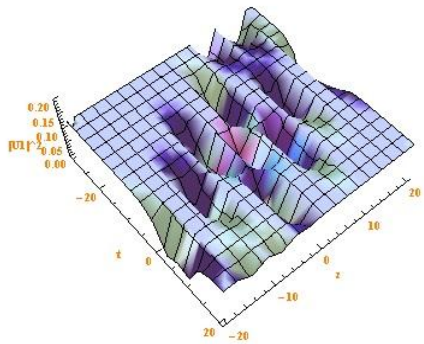


FIG.4. The parameters adopted here are c(z) = 2 Cos(z), e(z) = Cos(z), b(z) = 0.1 and f(z) = 1;

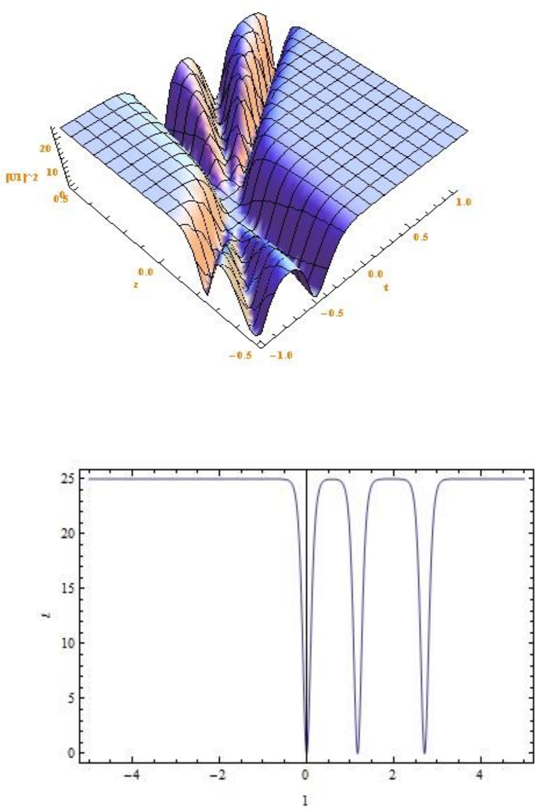


FIG. 6. The evolution plot of dark three-soliton solution with parameters are c(z) = 0.1, e(z) = 2.5, b(z) = 1 and f(z) = 1.5; (b) is the intensity plot corresponding to (a) derived from similar expression.

V. CONCLUSIONS

We have investigated that the integrable generalized variable-coefficient single component NLS system with higher order effects such as the third-order dispersion, self-steepening, and self-frequency shift, can be used to describe the propagation of intense electromagnetic field in multimode, inhomogeneous optical fiber media. We have derived the optical multi dark-soliton solutions via solving the Hirota's bilinear equations. Also, with symbolic computations we have showed that we are able to control the characteristics of optical multi-dark solitons in inhomogeneous optical fibers by choosing suitable variable-coefficient functions.

With models appearing in the previous papers[1],[7], it is notable that system (1) has three salient features: (i) soliton propagation (amplitude, velocity & acceleration) depends on b(z), c(z), e(z) and f(z); (ii) related coefficients are distributed functions of the variable z but not constants, and (iii) several higher order effects are included. That is just what makes our investigations important.

VI. ACKNOWLEDGEMENT

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